

Fluid flow and heat transfer around rectangular cylinders (the case of a width/height ratio of a section of 0.33 ~ 1.5)

T. IGARASHI

Department of Mechanical Engineering, The National Defense Academy, Yokosuka 239, Japan

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Abstract—Experimental investigations on the fluid flow and heat transfer around rectangular cylinders were carried out in the range $7.5 \times 10^3 \leq Re \leq 3.75 \times 10^4$. The width/height ratio (c/d) of the section ranged from 0.33 to 1.5. The local heat transfer coefficients around the cylinder become clear in connection with the flow characteristics. The overall heat transfer around the cylinder is given by $\overline{Nu}_m = C_m Re^{2/3}$ for $c/d \geq 0.67$. The average heat transfers on the side and the rear faces are given by $\overline{Nu}_s = C_s Re^{2/3}$ and $\overline{Nu}_b = C_b Re^{2/3}$, respectively. The constants C_m , C_s , and C_b increase with an increase in drag coefficient, C_D . The average heat transfer coefficient on the rear face, \overline{h}_b , is correlated with the r.m.s. value of the fluctuating pressure, Δp , by the expression $\overline{h}_b \propto \Delta p^{0.30-0.33}$.

1. INTRODUCTION

RECENTLY, the author carried out experimental studies on the heat transfer from a square prism to an air stream [1, 2]. In ref. [1], the average heat transfer coefficients at the angles of attack $\alpha = 0^\circ$ and 45° were as high as 40% of the values of Hilpert [3]. The result is so familiar as to be quoted in the textbook by Jakob [4]. In ref. [2], the local and average heat transfers on each face were made clear in connection with the flow characteristics [5] corresponding to the variation of flow patterns with the angle of attack.

The aerodynamic characteristics of a rectangular cylinder change drastically to about $c/d = 0.67$ and 2.8, when the width/height ratio (c/d) of the section of the cylinder increases systematically. This fact was found by Nakaguchi *et al.* [6] and subsequently confirmed by Bearman and Trueman [7]. Beyond $c/d = 2.8$, the shear layer separated from the cylinder reattaches periodically to the side face of the cylinder and the Strouhal number increases drastically. At $c/d = 0.67$ the position of vortex formation comes close to the rear face of the cylinder, then values of the drag coefficient and the minus base pressure coefficient have sharp peaks [6]. Recently, Okajima *et al.* [8] found that the Strouhal number varies discontinuously at about $c/d = 6.0$. They stated that at a ratio greater than 6.0, a stationary reattached flow follows, accompanied with a separated bubble.

Such a phenomenon as the shape of the cylinder, termed a critical geometry, has drawn attention to fluid mechanics. Regardless of its notice, little is known about the heat transfer around a rectangular cylinder. For that reason, the author has examined experimentally the characteristics of the local and average heat transfers from rectangular cylinders with a section of $c/d = 0.33 \sim 1.5$, and has related those to a previous paper [9] on characteristics of the flow around rectangular cylinders.

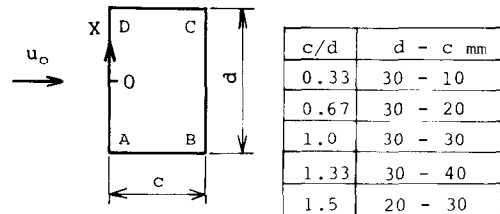


FIG. 1. Flow geometry.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The configuration of a rectangular cylinder is shown in Fig. 1. Experiments were performed in a low-speed wind tunnel with a working section 400 mm high, 150 mm wide and 800 mm long. The height of the section of the rectangular cylinder d was taken as 20 and 30 mm and the width/height ratio (c/d) of the section was varied from 0.33 to 1.5. These cylinders were 150 mm long and spanned the wind tunnel. The free stream velocity, u_0 , ranged from 6 to 20 m s^{-1} , and the turbulence intensity was 0.5% in this range. Therefore, the range of Reynolds numbers based on d was $7.5 \times 10^3 \leq Re \leq 3.75 \times 10^4$. The test cylinders used for measurement of heat transfer were made of acrylic resin, and the whole surface of the cylinders was covered with a stainless steel sheet 0.02 mm thick. By applying an alternating current from the electrodes attached on both sides of the cylinder, the cylinder was heated under a constant heat flux. The surface temperatures around the cylinder were measured with several copper-constantan thermocouples 0.1 mm in diameter buried in the circumference wall at 2 or 3 mm intervals and located immediately underneath the heater. In order to discuss the correlations between

NOMENCLATURE

C	constant	Re, Re_x	Reynolds numbers, $u_0 d/v, u_\infty x/v$
c, d	width and height of section of rectangular cylinder	S	Strouhal number
C_D	drag coefficient	u_0	free stream velocity
C_p, C_{pb}	pressure and base pressure coefficients	u_∞	velocity at the outer boundary layer
C'_p	fluctuating pressure coefficient, $\Delta p/0.5\rho u_0^2$	x	distance from forward stagnation point.
h, h_m	local and overall heat transfer coefficients	Greek symbols	
\bar{h}	average heat transfer coefficients on each face	λ	thermal conductivity of fluid
Nu, Nu_x	local Nusselt numbers, $hd/\lambda, h_x d/\lambda$	ν, ρ	kinematic viscosity and density of fluid.
$\overline{Nu}, \overline{Nu}_m$	average and overall Nusselt numbers, $\bar{h}d/\lambda, h_m d/\lambda$	Subscripts	
Δp	root-mean-square value of fluctuating pressure	b, f, s	rear, front and side faces
		r	rear stagnation point.

heat transfer on the rear face and fluctuating pressure, the r.m.s. fluctuating pressure was measured over the above range of Reynolds numbers. Furthermore, the experimental data on the Strouhal number, and time-mean and fluctuating pressure distributions were quoted from the previous report [9]. For $c/d = 1.0$, the results of heat transfer were also quoted from ref. [1].

3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1. Flow characteristics

As shown in Fig. 2, the Reynolds number has no influence on the Strouhal numbers of rectangular cylinders. Up to $c/d = 1.0$, the variation of the Strouhal number with c/d is not very much; $S = 0.15 \sim 0.125$. Beyond $c/d = 1.0$, the value decreases with an increase in c/d and has a minimum $S = 0.05$ at about $c/d = 2.8$. Figure 3(a) shows the

variations in base pressure coefficient at the center of the rear face and the drag coefficient of the cylinders with c/d . Both the coefficients are consistent with those of early papers [6, 7]. Each value of C_D and $-C_{pb}$ has a maximum at about $c/d = 0.67$. On the other hand, at $c/d = 2.8$, the two values do not present such a drastic change that the value of the Strouhal number increases stepwise from 0.04 to 0.16 at $c/d = 2.8$ [6]. Figure 3(b) shows the fluctuating pressure coefficient at the center of the rear face C'_p . The coefficient C'_p also increases abruptly and has a maximum at about $c/d = 0.67$. According to Nakaguchi *et al.* [6], at this time the location of vortex formation is much closer to the rear face. Examination of the flow visualization [9] reveals that the vortex is formed at an approximately identical location from the leading edge of the cylinder in the range of $0 \leq c/d \leq 0.7$. Beyond $c/d = 0.7$, the location moves downstream with an increase in c/d . As a consequence, the rolling up of shear layer becomes weak and the values of $-C_p$ and C'_p on the rear face decrease. In Figs. 3(a) and (b), it is noteworthy that the three values, C_D , $-C_{pb}$ and C'_p for $c/d = 1.0$, equal those for about $c/d = 0.2$, respectively. This fact suggests that the wake of a plate with $c/d = 0.2$ similar in characteristics to that of a square prism. Actually, close agreement between the heat transfer on the rear faces of the two was obtained [2].

The mean and fluctuating pressure distributions on the side and rear faces of rectangular cylinders are shown in Figs. 4 and 5. The ratio c/d has no influence on the pressure distributions on the front face. The coefficients C_p on the side and rear faces are affected by c/d and are kept constant along each face, respectively. While for $c/d \leq 0.67$, the two coefficients C_p on the side and rear faces almost agree, for $c/d \geq 1.0$ the

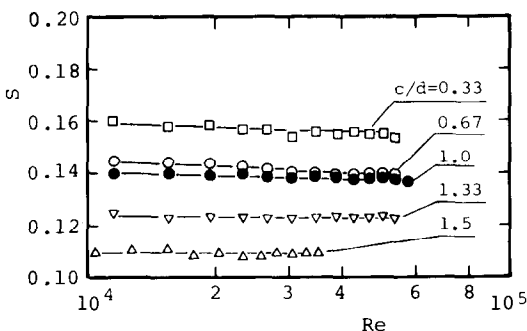


FIG. 2. Strouhal number.

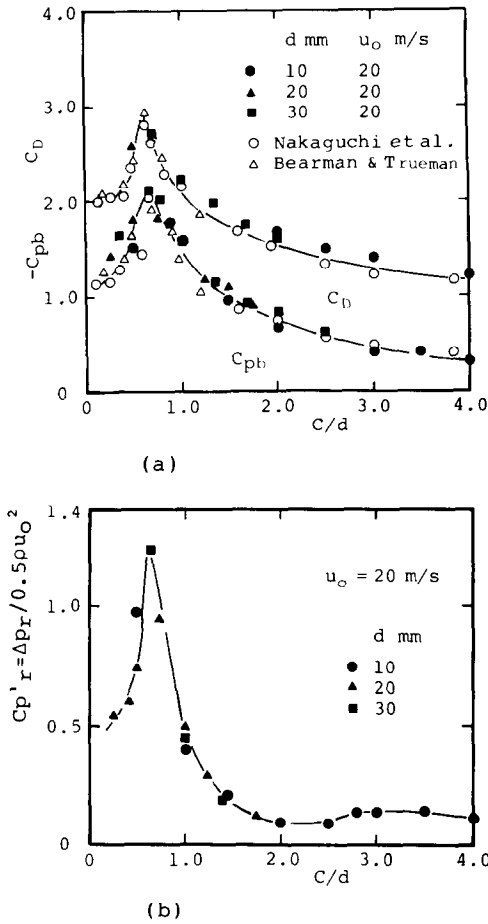


FIG. 3. Variations of flow characteristics with width/height ratio: (a) base pressure coefficient and drag coefficient; (b) fluctuating pressure coefficient.

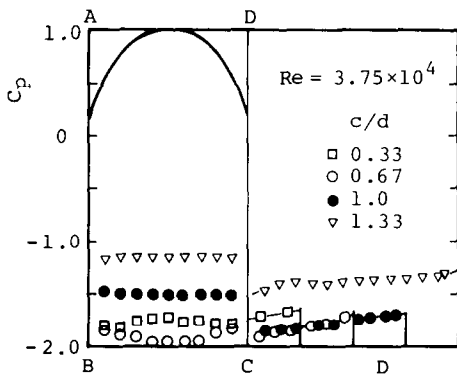


FIG. 4. Pressure distribution.

coefficient C_p on the side face is lower than that on the rear face.

The effect of the c/d ratio on the distribution of fluctuating pressure appears slightly on the front face, the coefficient C_p^1 increases with decreasing C_p . Such a tendency is remarkable on the rear face, that is, the value C_p^1 has a maximum at corners B and C, and it decreases toward the rear stagnation point. This is

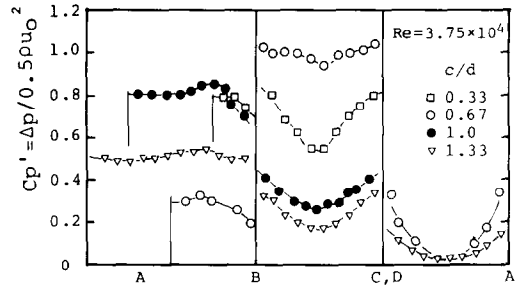


FIG. 5. Fluctuating pressure distribution.

because the shear layers from the front edge, corners D and A, roll up, then reattach on the rear face, corners B and C. For the critical geometry $c/d = 0.67$, the coefficient C_p^1 is unity over the whole rear face. Besides, the coefficient of C_p^1 on the side face is independent of that on the rear face.

In previous papers [1, 8, 9], it has been indicated that the average heat transfers on the rear face of a square prism and a circular cylinder are closely related to the r.m.s. fluctuating pressure. From the viewpoint mentioned above, the values Δp_r and Δp_c were obtained and demonstrated in Figs. 6(a) and (b), respectively. Except for $c/d = 0.67$ and 1.5, they can be presented by

$$\Delta p \propto u_o^{2.2}. \quad (1)$$

In the above two cases, the relations are given in the same form by $\Delta p \propto u_o^{2.0}$.

3.2. Local heat transfer

The accuracy of measurement of the local heat transfer performance from rectangular cylinders was examined. The laminar heat transfer on the front face of the cylinders was measured under the condition of a constant heat flux. In this case, the front face turned out to have a uniform surface temperature. It is well known that the local heat transfer coefficient at distance x from the forward stagnation point is given as

$$Nu_x / \sqrt{Re_x} = 0.57 Pr^{0.4} \quad (2)$$

where Re_x is defined as $u_\infty x / \nu$, and the velocity at the outer boundary layer u_∞ is determined on the basis of the foregoing pressure distributions. For instance, the experimental values obtained on the four models were compared with theoretical values. As is obvious from Fig. 7, these measurements are very accurate.

The variation of the distribution of local heat transfer coefficients around rectangular cylinders with $c/d = 0.33 \sim 1.5$ are shown in Figs. 8(a) and (b), respectively. On the front face, the heat transfer coefficients for $c/d = 0.33$ and 1.33 tend to increase from the stagnation point toward the leading edge, but such a tendency is obscured for the cases of $c/d = 0.67$ and 1.5. This cause cannot be explained. On the side face, the coefficient decreases rapidly from

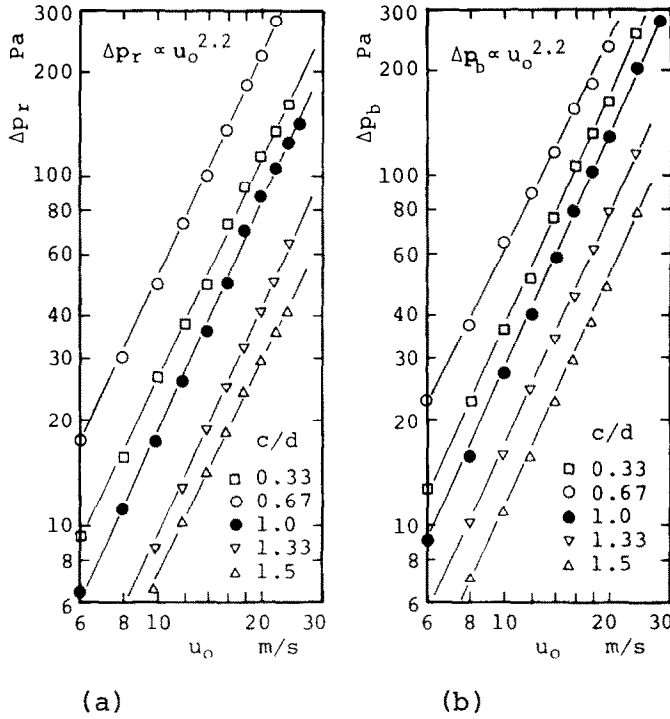


FIG. 6. Correlations of Δp_r and Δp_b vs u_0 .

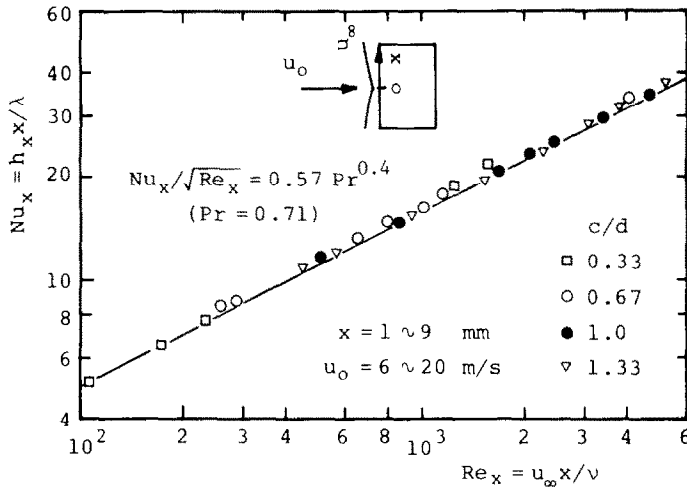


FIG. 7. Local coefficient of heat transfer in the neighborhood of the stagnation point on the front face: constant heat flux.

the trailing edge toward the leading edge, and has a minimum near corner A. For $c/d \geq 0.67$, the heat transfer coefficient near the leading edge corner on the side face is remarkably smaller than that on the front face. For $c/d = 0.33$ the coefficient near the trailing edge on the side face is relatively small compared with that on the rear face. The two coefficients are approximately equal for $c/d = 0.67$ and reverse the magnitude beyond $c/d = 1.33$. The distributions on the rear face remain constant except near the trailing edge. This tendency is remarkable for $c/d = 0.33$, which resembles a flat plate. The variation

of the local heat transfer coefficient on the rear face at $u_0 = 20 \text{ m s}^{-1}$ ($Re = 3.75 \times 10^4$) with c/d presents an adequate correlation with those of the characteristics of wake flow such as $-C_{pb}$, C_D and C_f' shown in Figs. 3(a) and (b). The larger these values the higher the heat transfer coefficient becomes.

A comparison of the distributions of the local heat transfer around rectangular cylinders with that around a square prism at $u_0 = 20 \text{ m s}^{-1}$ are shown in Fig. 9. For $c/d = 0.67$, the local heat transfer coefficients on the side and rear faces are larger than those of the square prism, respectively. On

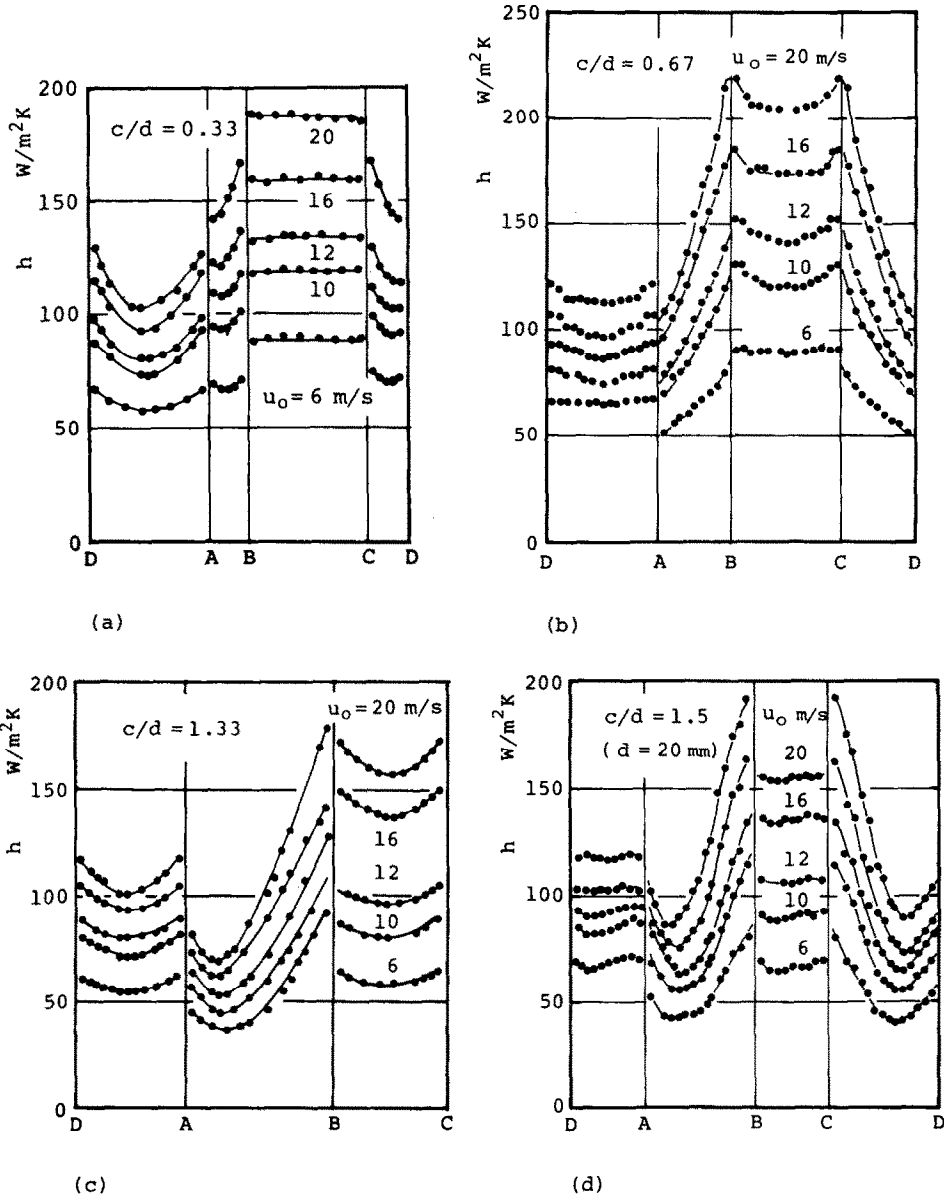


FIG. 8. Local coefficient of heat transfer around rectangular cylinders.

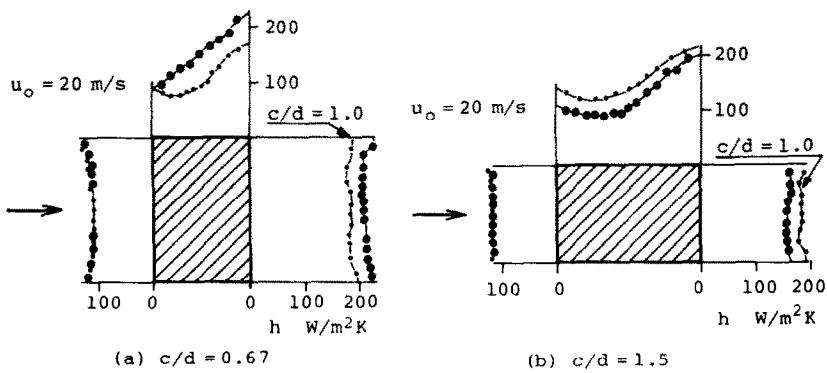


FIG. 9. Comparison of local heat transfer distributions around rectangular cylinders with a square prism.

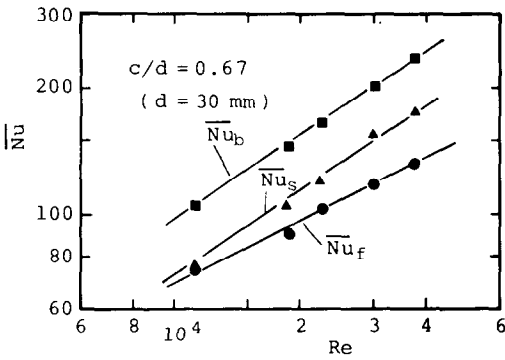


FIG. 10. Comparison of average heat transfer on every face.

the contrary, for $c/d = 1.5$ the relative magnitude is reversed. The heat transfer coefficients on the rear and side faces are correlated to values of $-C_{pb}$ and C_p . For $c/d = 0.67$, the position of vortex formation comes close to the rear face and the wake width has a minimum [9], then the fluid flow near the wake fluctuates violently. From the facts mentioned above, the heat transfer in the separated region increases at about $c/d = 0.67$. As the ratio c/d increases to 1.5, the heat transfer in the separated region decreases, because the position of vortex formation moves downstream, and the values of $-C_{pb}$ and C_p decrease.

3.3. Average heat transfer

The average heat transfer coefficient for every face has been examined. The result for $c/d = 0.67$ is shown in Fig. 10. The Nusselt number on the front face satisfies the same relation as that for a laminar boundary layer, $\overline{Nu}_f \propto Re^{1/2}$. The value of \overline{Nu}_f is lower than values of Nu_s and Nu_b on the side and rear faces. The Nusselt numbers on the side and rear faces satisfy the same relation as that for a separated region, $\overline{Nu}_s, \overline{Nu}_b \propto Re^{2/3}$. For the other c/d ratio, the dependence of Nusselt number on Reynolds number shows a similar tendency. For a large c/d ratio, the Nusselt number on the front face is higher than that on the side face. Actually, for $c/d = 1.0$, the Nusselt number on the side face is lower than that on the front face. For $c/d = 1.33$ and 1.50, the Nusselt numbers on the front face are higher than those on the side face up to high Reynolds numbers. Below $c/d = 1.5$, the Nusselt number on the rear face is considerably larger than those on the front and side faces.

Figures 11 and 12 show the variations in the average heat transfer on the side and rear faces with the ratio c/d . The average Nusselt numbers are given by

$$\text{side face: } \overline{Nu}_s = C_s Re^{2/3} \tag{3}$$

$$\text{rear face: } \overline{Nu}_b = C_b Re^{2/3} \tag{4}$$

The constants C_s and C_b for various c/d ratios are given in individual figures. While the constant C_s

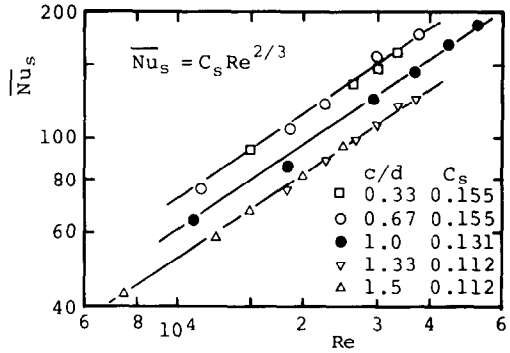


FIG. 11. Variations of average heat transfer on the side face with width/height ratio.

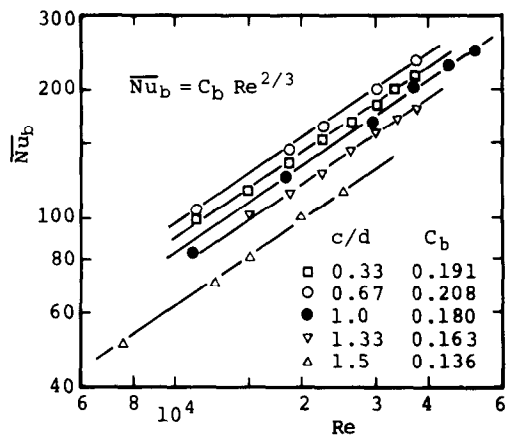


FIG. 12. Variations of average heat transfer on the rear face with width/height ratio.

decreases stepwise with an increase in c/d , the constant C_b has a maximum at $c/d = 0.67$, and then decreases with an increase in c/d . The variation of the value of C_b corresponds to the characteristics of the flow shown in Figs. 2 and 3. The overall Nusselt number, \overline{Nu}_m , is indicated in Fig. 13. From the figure, the following correlations are obtained

$$\overline{Nu}_m = C_m Re^{2/3} \quad (c/d \geq 0.67) \tag{5}$$

$$\overline{Nu}_m = 0.25 Re^{0.62} \quad (c/d = 0.33). \tag{6}$$

The variations of the above constants C_s, C_b and C_m with c/d are plotted in Fig. 14. In the figure the results of the flat plate with $d = 50$ mm, for $c/d = 0.14$ (ref. [10]) are also illustrated for reference. The variation in the constant C_b on the rear face is similar to those of the flow characteristics in the separated region mentioned above.

3.4. Correlation between heat transfer and fluctuating pressure in separated region

In ref. [2], it has been reported that the average heat transfer on the rear face of the square prism is correlated to the r.m.s. fluctuating pressure at the rear stagnation point, Δp_r . A similar relationship was also

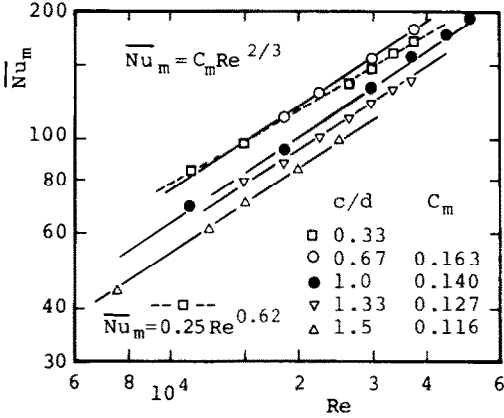


FIG. 13. Overall heat transfer coefficient.

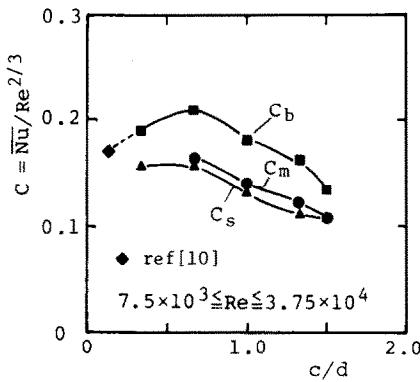


FIG. 14. Overall and average heat transfer on the side face and the rear face with width/height ratio.

obtained for the cases for local heat transfer at the rear stagnation point, h_r , and the average heat transfer on the rear face of a circular cylinder, \bar{h}_b [8, 9]. The value of the r.m.s. fluctuating pressure, Δp_r , is one of the most dominant factors of the heat transfer in the separated region. From this point of view, the average heat transfer on the rear face of the rectangular cylinders is arranged by using the r.m.s. fluctuating pressure at the rear stagnation point, Δp_r . The result obtained is demonstrated in Fig. 15

$$\bar{h}_b = C \Delta p_r^{0.30 \sim 0.33} \quad (7)$$

Equation (7) can be rearranged by dimensionless

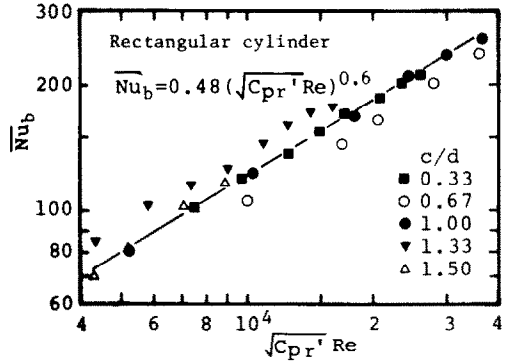


FIG. 16. Correlation of average heat transfer vs the fluctuating pressure coefficient on the rear face of a rectangular cylinder.

groups, that is, \bar{Nu}_b , Re and C_p' , as shown in Fig. 16. The relation is formulated as

$$\bar{Nu}_b = 0.48(\sqrt{C_p' Re})^{0.6} \quad (8)$$

4. CONCLUSIONS

Experimental investigations on the local heat transfer from rectangular cylinders to an air stream were carried out in the range $7.5 \times 10^3 \leq Re \leq 3.75 \times 10^4$. The width/height ratio (c/d) of the section ranged from 0.33 to 1.5. The local and average heat transfer performances on every face are clarified in connection with the characteristics of the flow around the cylinder. The main results are summarized below.

(1) The average heat transfer coefficients on the side and rear faces of the rectangular cylinder are expressed as follows, regardless of the c/d ratio

$$\begin{aligned} \bar{Nu}_s &= C_s Re^{2/3} \\ \bar{Nu}_b &= C_b Re^{2/3} \end{aligned}$$

For $c/d \geq 0.67$ the overall heat transfer coefficient satisfies a similar relation

$$\bar{Nu}_m = C_m Re^{2/3}$$

(2) Constants C_s , C_b and C_m vary with the variation of the value of the drag coefficient, C_D .

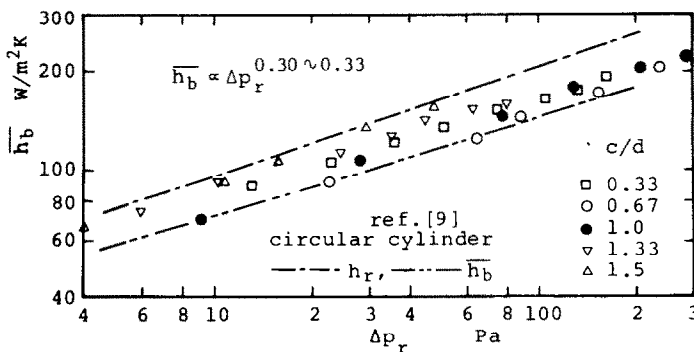


FIG. 15. Correlation of average heat transfer on the rear face vs Δp_r .

(3) The individual variations of the above constants C_s , C_b and C_m with the c/d ratio have a maximum at the critical geometry, $c/d = 0.67$.

(4) The heat transfer coefficient on the rear face of the cylinder is higher than those on the other faces. Although the coefficient on the side face is generally higher, it is lower than that on the front face in the range of low Reynolds number for $c/d \geq 1.33$.

(5) The average heat transfer coefficient on the rear face is related to the r.m.s. value, Δp_r , of the fluctuating pressure at the center of the face by the expression

$$\bar{h}_b \propto \Delta p_r^{0.30 \sim 0.33}.$$

(6) The above correlation is formulated by dimensionless parameters as

$$\overline{Nu}_b = 0.48(\sqrt{C_p Re})^{0.6}.$$

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ÉCOULEMENT ET TRANSFERT THERMIQUE AUTOUR DE CYLINDRES RECTANGULAIRES

Résumé—Des recherches expérimentales sur l'écoulement et le transfert thermique autour de cylindres rectangulaires sont conduites dans le domaine $7,5 \cdot 10^3 \leq Re \leq 3,75 \cdot 10^4$. Le rapport largeur/hauteur (c/d) de la section varie entre 0,33 et 1,5. Les coefficients de transfert locaux autour du cylindre sont reliés aux caractéristiques de l'écoulement. Le transfert global autour du cylindre est donné par $\overline{Nu}_m = C_m Re^{2/3}$ pour $c/d \geq 0,67$. Les transferts moyens sur les faces latérales et arrière sont données par $\overline{Nu}_s = C_s Re^{2/3}$ et $\overline{Nu}_b = C_b Re^{2/3}$. Les constantes C_m , C_s et C_b augmentent en même temps que le coefficient de trainée C_D . Le coefficient moyen de transfert sur la face arrière \bar{h}_b est relié à la valeur r.m.s. de la pression fluctuante Δp_r par l'expression $\bar{h}_b \propto \Delta p_r^{0.30 \sim 0.33}$.

STRÖMUNG UND WÄRMEÜBERGANG UM RECHTWINKLIGE ZYLINDER (DER EINFLUSS DES BREITEN-HÖHEN-VERHÄLTNISSSES 0.33 BIS 1.5)

Zusammenfassung—Es werden experimentelle Untersuchungen zur Strömung und zum Wärmeübergang an rechtwinkligen Zylindern im Bereich $7,5 \times 10^3 \leq Re \leq 3,75 \times 10^4$ durchgeführt. Das Breiten-Höhen-Verhältnis c/d variiert von 0,33 bis 1,5. Die örtlichen Wärmeübergangskoeffizienten um den Zylinder sind in Verbindung mit den Strömungscharakteristiken zu erklären. Der Gesamtwärmeübergangs-Koeffizient am Zylinder läßt sich für $c/d \geq 0,67$ durch $\overline{Nu}_m = C_m Re^{2/3}$ korrelieren, die mittleren Wärmeübergangskoeffizienten an den seitlichen und rückwärtigen Flächen durch $\overline{Nu}_s = C_s Re^{2/3}$ bzw. $\overline{Nu}_b = C_b Re^{2/3}$. Die Konstanten C_m , C_s und C_b nehmen mit dem Formwiderstandsfaktor C_D zu. Der mittlere Wärmeübergangskoeffizient an der Rückseite \bar{h}_b wird mit dem Effektivwert der Druckschwankungen Δp_r durch den Ausdruck $\bar{h}_b \sim \Delta p_r^{0.30 \sim 0.33}$ korreliert.

ТЕПЛООБМЕН И ОБТЕКАНИЕ ЖИДКОСТЬЮ ПРЯМЫХ ЦИЛИНДРОВ

Аннотация—Экспериментально изучено течение жидкости и теплообмен на поверхности прямых цилиндров при $7,5 \cdot 10^3 \leq Re \leq 3,75 \cdot 10^4$. Отношение высоты к диаметру (c/d) изменялось в диапазоне от 0,33 до 1,5. Коэффициенты локального теплообмена поверхности цилиндра находятся из характеристик течения. Суммарный теплообмен цилиндра задан как $\overline{Nu}_m = C_m Re^{2/3}$ для $c/d \geq 0,67$. Осредненный теплообмен на торцах цилиндра определялся как $\overline{Nu}_s = C_s Re^{2/3}$ и $\overline{Nu}_b = C_b Re^{2/3}$, соответственно. Постоянные C_m , C_s и C_b возрастают с увеличением коэффициента сопротивления C_D . Осредненный коэффициент теплопереноса на заднем торце связан со среднеквадратичной величиной флуктуации давления Δp соотношением $\overline{h}_b \propto \Delta p^{0,3 \sim 0,33}$.